Christian Heritage School
2022 Required Summer Review Work
for all students enrolling in Honors Precalculus
(based on content studied in Algebra 2 Common Core from Pearson)

Objective: Summer review will help students to review and practice skills needed in preparation for Honors Precalculus.

This assignment contains a summary of prerequisite concepts and sample problems for all students enrolling in Honors Precalculus for Fall 2022 to complete. The work is organized by concept, with some sample problems for each. *Students could potentially be quizzed on this assignment in early September.*

There are some great resources available online if you are stuck or in need of more practice. For example, Khan Academy has many resources for Algebra studies.

Please complete the entire assignment and check your answers, which are included at the end. Graphs can be checked using a graphing calculator or Desmos.

1. Absolute value equations and inequalities
    a. Remember that there are two parts to the solution. Get the absolute value by itself first, then make the "positive" equation and the "negative equation". OR you can use the distance idea: $|x - 7| = 4$ reads "the distance that x is from 7 is 4 units"; meaning $x = 11$ or 3
    b. Absolute value inequalities: "land – gor" means less than is "and" and greater than is "or".
    c. $3|x + 10| = 6$
    d. $4|y - 9| > 36$

2. Relations and Functions
    a. A relation is a function if every input is paired with exactly one output (no repeat of inputs). A function is one-to-one if it (graphically) passes the horizontal line test or (numerically) if there is no repeat of outputs. Be able to discuss domain, range, and end behavior. (NOTE: the preferred end behavior description for H Pre Calc is As $x \rightarrow +\infty$, $y \rightarrow$ ...)
    b. Is the following relation a function? Find its domain and range. If it is a function, is it one-to-one? {(10, 2), (-10, 2), (6, 4), (5, 3), (-6, 7)}

3. Linear functions and slope-intercept form
    a. Slope-intercept form $y = mx + b$
    b. Point-slope form $y - y_1 = m(x - x_1)$
    c. Standard form: $Ax + By = C$

d. Parallel lines (same slope) and perpendicular lines (negative reciprocal slope)
e. Write an equation of the line in standard form with slope 4 passing through the point (-2, -5)
f. Write an equation of the line perpendicular to $x + 2y = 6$ through (8, 3)

4. Transformations of functions
    a. Parent function: $y = f(x)$
    b. Vertical translation: $y = f(x) + k$
    c. Horizontal translation: $y = f(x – h)$
    d. Reflection in the x-axis: $y = -f(x)$
    e. Reflection in the y-axis: $y = f(-x)$
    f. Vertical stretch/compression: $y = a \cdot f(x)$
    g. Write the equation for the transformation of the graph of $y = f(x)$ that is translated 2 units left and 7 units down
    h. Write the equation for the transformation of the graph of $y = f(x)$ that is translated 5 units right and reflected across the x-axis
    i. Write the equation for the transformation of the graph of $y = f(x)$ that is translated 3 units up and reflected across the y-axis
    j. Describe the transformation(s) to the parent function, f(x), if $g(x) = f(x) – 4$
    k. Describe the transformation(s) to the parent function, f(x), if $h(x) = 12f(x) + 2$
    l. Describe the transformation(s) to the parent function, f(x), if $k(x) = -2f(-x)$

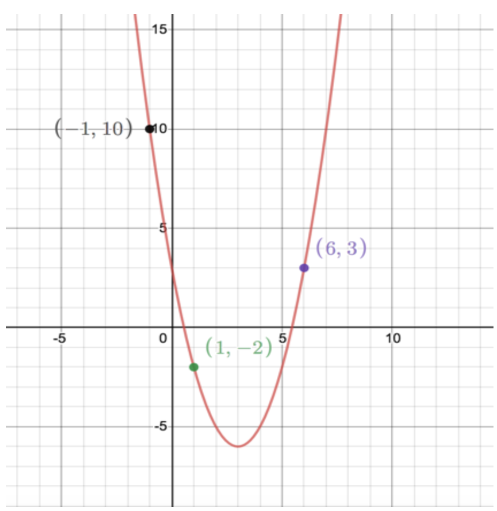
5. Absolute value functions and graphs
    a. $Y = |x|$ is the parent function
    b. Transformations from above section #4 can be applied
    c. Key ideas for graphing: vertex (h, k); opens up/down, depending if $a > 0$ or $a <0$; vertical stretch corresponds to "slope"; shape is a V
    d. Graph $f(x) = -2|x – 5|$
    e. Without graphing, identify the vertex and axis of symmetry of $y = 2|x – 4|$

6. Solving systems of equations efficiently
    a. Solving by graphing: finding the point of intersection (learn how to do this on your calculator! Use INTERSECT option in CALC menu)
    b. Solving by substitution and/or elimination (systems of two)
    c. Solving systems of 3 equations: the key is to substitute for or eliminate the same variable twice! (or use matrices!)
    d. Also review how to solve systems of 3 equations using matrices on your graphing calculator, as it is the most time-efficient method.
    e. Solve using a graphing calculator: $3x + 2y = 4$
       $2x – 4y = 8$
    f. (suggested to use elimination/matrices)
       Solve $x + y – 2z = 8$
       $5x – 3y + z = -6$
       $-2x – y + 4z = -13$

g. (suggested to use substitution/matrices)
Solve $3x + y - 2z = 22$
$x + 5y + z = 4$
$x = -3z$

h. (suggested to use substitution/matrices)
Solve $x + 2y + z = 14$
$y = z + 1$
$x = -3z + 6$

7. Quadratic functions and transformations
   a. $Y = x^2$ is a parabola; this parent function has vertex (0, 0); opens up; has next points up one and over one; third set is up three and over one; there is symmetry across its vertex (line is the axis of symmetry)
   b. We described its transformations in vertex form: $y = a(x - h)^2 + k$, where (h, k) is the vertex. All of the transformations in section 4 above apply to this graph.
   c. To find the equation of a quadratic function given the vertex and a point, substitute the vertex into the formula for h and k; substitute the point in for x and y; solve for a.
   d. Graph $y = -3(x + 7)^2 - 8$; then identify the vertex and axis of symmetry.
   e. Write the equation of the parabola whose vertex is (1, 2) that passes through the point (2, -5). Give your answer in vertex form.
   f. Find the quadratic equation in standard form of the parabola shown in the Desmos graph below by first writing and efficiently solving a system of three equations. (Hint: plug each of the three points into $y = ax^2 + bx + c$ to create your system of three equations).



8. Quadratic functions in standard form

a. Y = $ax^2 + bx + c$; a tells us whether the graph opens up or down; (0, c) gives us the y-intercept; x = $-\frac{b}{2a}$ gives us the x-coordinate of the vertex and the axis of symmetry; to find the y-coordinate, substitute the x-coordinate into the equation.
b. To graph, find the vertex (above); find the y-intercept (above); use symmetry to plot the third point. OR, you can always make a table of values; just be sure to use x-values on both sides of the vertex
c. Graph $y = x^2 + 2x + 3$ by finding the y-intercept and the vertex.
d. Identify the vertex, axis of symmetry, maximum or minimum value, and range of the parabola $y = -x^2 + 2x + 5$
e. Apply: A model for a company's revenue from selling a software package is $R = -2.5p^2 + 500p$, where p is the price in dollars of the software. What price will maximize revenue? Find the maximum revenue.

9. Factoring quadratic expressions
    a. Factoring is rewriting an expression as a product of factors; in other words, it's the reverse of foiling or the box problem.
    b. Always remember to begin by factoring out the GCF. If the highest powered term is negative, factor out the negative to make the rest of the problem easier.
    c. Keep an eye out for the special patterns: difference of two squares and perfect square trinomials.
    d. Remember that not every expression is factorable.
    e. Factor $x^2 – 4$
    f. Factor $k^2 – 18k + 81$
    g. Factor $3x^2 + 31x + 36$
    h. Factor $81y^2 + 49$
    i. Factor $4x^2 + 16x + 8$
    j. $3t^2 – 24t$
    k. * challenge: $(x + 3)^2 + 3(x+3) – 54$ [HINT: let y = x + 3]

10. Quadratic Equations - solving efficiently
    a. To solve quadratic equations by factoring, gather all terms on one side, factor (as in section #9 above), and then set each factor equal to zero and solve.
    b. To solve quadratic equations by graphing, use a graphing calculator or utility, and find the solutions by finding the zeros of the function (the ZERO option in the CALC menu).
    c. Vertical motion problems: $h(t) = -16t^2 + v_0t + h_0$, where h represents the height of an object at time t; v_0 represents the initial (upward) velocity of the object, and h_0 represents the initial height above the ground.
    d. Solve by factoring: $x^2 + 18 = 9x$
    e. Solve by graphing: $3x^2 – 5x – 4 = 0$
    f. Using the vertical motion function $h(t) = -16t^2 + 1700$, what does the constant 1700 tell you about the height of the object? What does the coefficient of t^2 tell

you about the direction of motion? When will the object be 940 feet above the ground? What is a reasonable domain and range for the function h?

g. Solve $5x^2 = 80$ by finding square roots.
h. Solve $(x - 1)^2 = 5$ by finding square roots.

11. Completing the Square
    a. Find "c" to complete the square (create a perfect square trinomial): $x^2 + 20x +$ __
    b. Find "c" to create a PST: $x^2 - x +$ __
    c. To solve by completing the square: $x^2 + 8 - 10x = 0$
        i. Get the constant by itself $x^2 - 10x = -8$
        ii. Divide through every term by "a" ---
        iii. Take the new "b" and divide by 2 $-10/2 = -5$
        iv. Square it $(-5)^2 = 25$
        v. Add that number to both sides $x^2 - 10x + 25 = -8 + 25$
        vi. Factor the left side; add the right $(x - 5)^2 = 17$
        vii. Square root both sides $x - 5 = \pm\sqrt{17}$
        viii. Solve for x $x = 5 \pm \sqrt{17}$
    d. Solve by completing the square: $x^2 + 4x + 2 = 0$
    e. $x^2 - 2x = 5$
    f. $9x^2 - 12x - 2 = 0$

12. Quadratic formula
    a. Uses standard form: $ax^2 + bx + c = 0$
    b. $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
    c. Discriminant: $b^2 - 4ac$, the part under the radical; determines number and type of solutions
        i. If discriminant = 0, there is one real number solution
        ii. If discriminant > 0, there are two real number solutions
        iii. If discriminant < 0, there are two complex number solutions (containing *i*)
    d. Solve using the QF: $x^2 - 5x - 7 = 0$
    e. Solve using the QF: $2x^2 - 5x = 3$
    f. Find the discriminant and determine the number of real solutions to $x^2 = 2x - 9$

13. Complex numbers
    a. Standard form; a + b*i*; $i = \sqrt{-1}$; $i^2 = -1$
    b. To add or subtract, combine the real portions (a) and the imaginary portions (b)
    c. To multiply, use FOIL or box problem and simplify
    d. To divide, multiply top and bottom by conjugate of the denominator. For example, given $\frac{2}{4-i}$, multiply top and bottom by 4 + i and simplify.
    e. Simplify $\sqrt{-12}$
    f. Find (5 – 3i) – (-2 + 4i)
    g. Find (4 + 3i)(-1 – 2i)

h. Find (-6 + i)(-6 – i)
i. Find the quotient $\frac{2+3i}{1-4i}$

14. Use your calculator to solve quadratic systems (INTERSECT option in CALC menu – do NOT use "trace"):
    a. $Y = -x^2 - x + 12$
       $Y = x^2 + 7x + 12$

15. Simplifying radical expressions
    a. Break the radical down into the "can/cannot be rooted" pieces
    b. Use | | symbols when taking an even root and the exponent comes out odd, such as $\sqrt{36x^6} = 6|x^3|$
    c. Simplify $\sqrt{0.49}$
    d. Simplify $\sqrt{81x^2}$
    e. Simplify $\sqrt[3]{125x^6y^9}$

16. Binomial radical expressions
    a. Addition/subtraction: similar to "like terms" - $\sqrt{3} + \sqrt{2}$ cannot be combined
    b. Multiplication: a foil/box problem
    c. Division: rationalize the denominator by multiplying top and bottom by the conjugate
    d. Simplify: $(5 - 4\sqrt{5})(-2 + \sqrt{5})$
    e. Simplify: $\frac{5}{-3-3\sqrt{3}}$
    f. Simplify: $2\sqrt{6} - 2\sqrt{24}$

17. Solving radical equations
    a. Isolate the radical; solve for the variable; CHECK for extraneous solutions
    b. Solve: $\sqrt{v+3} - 1 = 7$
    c. Solve: $r = \sqrt{-1-2r}$

18. Rational Roots Theorem
    a. Factors of the constant divided by factors of the leading coefficient
    b. List all the possible rational roots of $f(x) = 5x^4 + 32x^2 - 21$

19. Solving polynomial equations by factoring
    a. Sum of two cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
    b. Difference of two cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
    c. Factor: $8x^3 + 27$
    d. Factor: $64x^3 - 1$

e. Solve by factoring: $x^4 - 3x^2 - 4 = 0$

20. Synthetic division and the Remainder theorem
    a. Given $P(x) = -x^3 + 6x - 7, find\ P(2)$.

21. Finding zeros of polynomial functions
    a. First you can list out the possible rational roots and begin to test them;
    b. OR use your calculator to find real zeros from the graph; then synthetically divide them out to "reduce" the equation to a quadratic. Solve for remaining zeros using the quadratic formula.
    c. Find all the zeros of $f(x) = x^4 + x^3 - 7x^2 - 9x - 18$

ANSWERS to non-graphing calculator questions:
1c. x = -8, x = -12
1d. y < 0 or y > 18
2b. yes; D: {-10, -6, 5, 6, 10}; R: {2, 3, 4,7}; not one-to-one
3e. 4x – y = -3
3f. y = 2x – 13
4g. y = f(x + 2) – 7
4h. y = -f(x – 5)
4i. y = f(-x) + 3
4j. translated 4 units downward
4k. vertical stretch by factor of 12, translated 2 units up
4l. vertical stretch by factor of 2; reflected across the x-axis; reflected across the y-axis
5e. vertex: (4, 0); aos: x = 4
6d. (2, -1)
6e. (1, 3, -2)
6f. (6, 0, -2)
6g. no solution (think about why!)
7d. vertex (-7, -8); aos: x = -7
7e. $y = -7(x - 1)^2 + 2$
7f. $y = x^2 - 6x + 3$
8d. vertex (1, 6); aos: x = 1; max: 6; range: $y \leq 6$
8e. $100; $25,000
9e. (x + 2)(x – 2)
9f. $(k - 9)^2$
9g. (3x + 4)(x + 9)
9h. not factorable
9i. $4(x^2 + 4x + 2)$
9j. 3t(t – 8)
9k. (x + 12)(x – 3)
10d. x = 3, 6

10f. initial height of object; accelerating downward; 6.89 s; D: $0 \leq t \leq 10.4$ (why is the domain not all real numbers?); R: $0 \leq h \leq 1700$
10g. $x = \pm 4$
10h. $x = 1 \pm \sqrt{5}$
11a. 100
11b. ¼
11d. $-2 \pm \sqrt{2}$
11e. $1 \pm \sqrt{6}$
11f. $\frac{2}{3} \pm \frac{\sqrt{6}}{3}$

12d. $\frac{5}{2} \pm \frac{\sqrt{53}}{2}$
12e. 3, -½
12f. -32; no real number solutions
13e. $2i\sqrt{3}$
13f. $7 - 7i$
13g. $2 - 11i$
13h. 37
13i. $\frac{-10}{17} + \frac{11}{17}i$ (must be separated into a + bi)
15a. 0.7
15b. $9|x|$
15c. $5x^2y^3$
16d. $-30 + 13\sqrt{5}$
16e. $\frac{5-5\sqrt{3}}{6}$
16f. $-2\sqrt{6}$
17b. v= 61
17c. no solution
18b. $\pm\{1, 3, 7, 21, 1/5, 3/5, 7/5, 21/5\}$
19c. $(2x + 3)(4x^2 - 6x + 9)$
19d. $(4x - 1)(16x^2 + 4x + 1)$
19e. $x = \pm 2$ and $\pm i$
20a. P(2) = -3
21c. $x = \pm 3 \; and \; \frac{-1}{2} \pm \frac{i\sqrt{7}}{2}$